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Earth Albedo Input to Flat Plates

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In this short note, the results of an analysis considering the earth reflected solar radiation incident upon a spinning flat plate are presented briefly. A general description of the problem is given, as well as a definition of all of the geometrical parameters, even though the final result itself is not given explicitly. However, the final integral expression is given, as well as the expressions for determining the integration limits. In actual practice, it proves to be rather easy to perform the integration with the aid of a computer. The parameters introduced succeed in defining the orientation of the surface with respect to the earth. No attempt is made here to give these parameters in terms of orbital parameters. Even so, unfortunately, it would not relieve the reader from the troublesome task of determining the remainder of the parameters from analysis of such data as time of launch, point of launch, injection angle, etc., for any particular problem that he may wish to consider.

Nomenclature

S = mean solar constant
$\mathbf{s}$ = solar vector
α = mean albedo of earth
$\mathbf{r}$ = vector between plate and earth's center
$\mathbf{A}$ = normal to plate
ϑ_s = angle between $\mathbf{s}$ and $\mathbf{r}$
λ = angle between $\mathbf{A}$ and $\mathbf{r}$
φ = azimuthal coordinate of $d\Sigma$
ξ = angle between $d\Sigma$ and $-\boldsymbol{\rho}$
$d\Sigma$ = element of terrestrial surface area
$\boldsymbol{\rho}$ = vector between plate and $d\Sigma$
β = angle between $-\mathbf{s}$ and $d\Sigma$
ϑ = colatitudinal coordinate of $d\Sigma$
η = angle between $\mathbf{A}$ and $\boldsymbol{\rho}$
r_0 = radial coordinate of $d\Sigma$, $\equiv 1$
σ = angle between $\mathbf{r}$ and $\boldsymbol{\rho}$

Introduction

In a recent paper,[1] the problem of determining the solar radiation reflected by the earth which is incident upon a spherical satellite has been discussed. In this note, the author proposes to consider (on the basis of the model used before, i.e., that the earth is a uniform, diffuse reflector) the energy input to an arbitrarily oriented flat plate. The problem is essentially one of geometry, and the author defines and gives expressions for all those parameters necessary for the description of the problem, as well as the general expression that must be integrated. Apart from the obvious application of the methods to determine the total power input to a satellite component such as a solar cell array, the results of the work as presented herein lend themselves to determining the energy distribution upon any of the surfaces of a satellite or space vehicle for which the orientational parameters can be given. Since these vehicles, in general, are spinning, the incident energy determination, to be useful, must be averaged over a spin period. This precludes treatment of surfaces that undergo shielding by other members of the satellite during a portion of each spin period, unless the values of the spin angle at which the surface is eclipsed can be determined. Since knowledge of the spin shadow parameters makes it possible to treat cases of partial shielding by making only simple modifications in the results, these will not be treated here. However, as a result of the spin, many surfaces do suffer self-shielding (present their backsides, as it were) during a portion of each revolution, and these cases are treated. Incidentally, a satellite surface spinning about an axis passing through its center presents the same physical picture to the earth if it spins about an axis parallel to the first but displaced from it. Therefore, no further mention is made of either case. In this work, the actual solution of the problem is left to an IBM 7090 computer. Since the

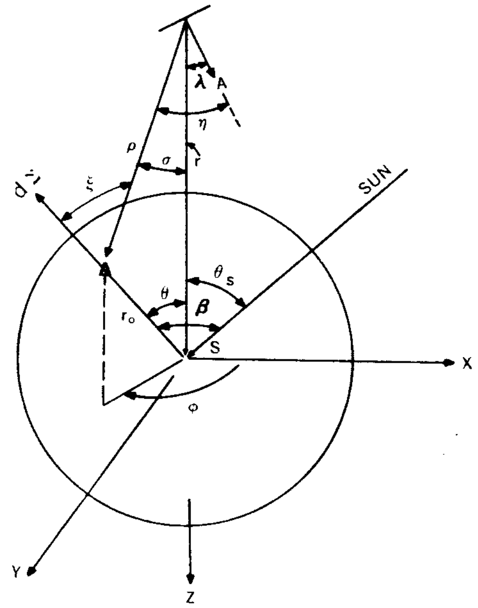

Fig. 1 Earth-satellite geometry for the general case

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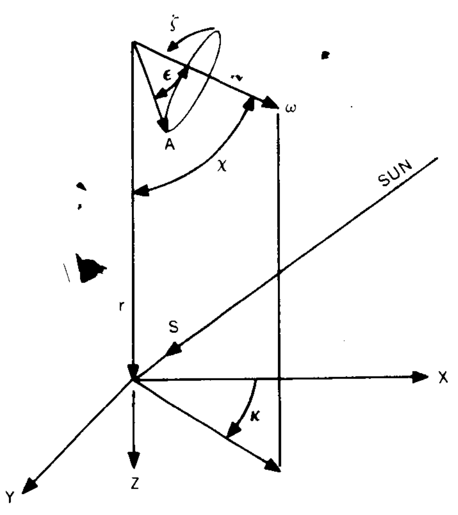

Fig. 2 Definition of the spin parameters

number of parameters involved is rather large, it is impractical to attempt to present here the results of any sample calculations either graphically or in tabular form.

Analysis

In Fig. 1, the needed parameters are defined, and the geometry is delineated. The associated definitions are given in the nomenclature.

The general expression for the incident reflected radiation to a plate of unit area is

$$P = \int_\Sigma \frac{S\alpha \cos\beta}{\pi} \cos\xi \frac{\cos\eta}{\rho^2} d\Sigma \tag{1}$$

In Eq. (1), $S\alpha \cos\beta d\Sigma$ gives the amount of incident solar energy reflected by $d\Sigma$. This quantity multiplied by $\cos\xi/\pi$ gives the amount of energy reflected by $d\mathbf{\Sigma}$ in the direction of the plate per unit solid angle. The factor $\cos\eta/\rho^2$ gives the solid angle subtended at $d\mathbf{\Sigma}$ by the plate of unit area. Equation (1) becomes[1, 2]

$$P = \frac{S\alpha}{\pi} \int_\vartheta \int \frac{(r\cos\vartheta - 1)(\cos\vartheta \cos\vartheta_s + \sin\vartheta \sin\vartheta_s \cos\varphi)}{(r^2 + 1 - 2r\cos\vartheta)^{3/2}} \cos\eta \sin\vartheta \, d\vartheta d\varphi \tag{2}$$

In Eq. (2), the range of the ϑ integration is $0 \leqslant \vartheta \leqslant \vartheta_m$, where $\vartheta_m = \cos^{-1}(1/r)$, and the range of the ϑ integration is $0 \leqslant \varphi \leqslant \varphi_m$. The determination of φ_m is not quite so simple. Upon some reflection, it can be seen that φ_m is determined by the fact that the side of the plate in question no longer receives any reflected solar radiation from a particular element of surface area $d\mathbf{\Sigma}$, the azimuthal coordinate of which is φ, and that this situation comes about in two ways: 1) the source function $S\alpha \cos \beta d\Sigma$ goes to zero, and 2) the solid angle term $\cos\eta/\rho^2$ goes to zero. The first of these is found by setting $\cos\beta = 0$, from which one has [Ref. 1, Eq. (5)]

$$\varphi_{m1} = \cos^{-1}(-\cot\vartheta \cot\vartheta_s) \tag{3}$$

and the second by equating $\cos\eta$ to zero. However, before this can be done, it becomes necessary to introduce several new parameters and to define the value of $\cos\eta$ in terms of these.

At this point the following new quantities are introduced:

$\boldsymbol{\omega}$ = satellite spin axis (assumed to coincide with its symmetry axis); it coincides with and has same direction as the angular momentum vector

χ = the angle between $\mathbf{r}$ and $\boldsymbol{\omega}$

ϵ = the angle between $\boldsymbol{\omega}$ and $\mathbf{A}$ (a fixed quantity for most satellites)

ζ = azimuthal angle of spin of $\mathbf{A}$ about $\boldsymbol{\omega}$

κ = the angle of rotation of $\boldsymbol{\omega}$ about $\mathbf{r}$, which gives the orientation of $\boldsymbol{\omega}$ with respect to the $\mathbf{s}$, $\mathbf{r}$ plane (defined similar to φ)

In Fig. 2, these quantities are shown, and a coordinate system (X,Y,Z) is defined, from which φ and κ are defined. As $\mathbf{A}$ rotates about $\boldsymbol{\omega}$, it coincides with the $\boldsymbol{\omega}$, $\mathbf{r}$ plane on two occasions. The zero value of ζ is taken to be the situation for which $\mathbf{A}$ lies in the $\boldsymbol{\omega}$,$\mathbf{r}$ plane, and its motion is in the positive κ direction. Then,

$$\begin{aligned}\cos\eta = {} & \frac{(r - \cos\vartheta)}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}} (\cos\chi \cos\epsilon - \sin\chi \sin\epsilon \cos\zeta) + \\ & \frac{\sin\vartheta}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}} [\cos\varphi \cos\kappa \cos\epsilon \sin\chi + \\ & \sin\varphi \sin\kappa \cos\epsilon \sin\chi + \cos\varphi \cos\kappa \sin\epsilon \cos\chi \cos\zeta + \\ & \sin\varphi \sin\kappa \sin\epsilon \cos\chi \cos\zeta + \sin\varphi \cos\kappa \sin\epsilon \sin\zeta - \\ & \cos\varphi \sin\kappa \sin\epsilon \sin\zeta]\end{aligned} \tag{4}$$

Setting the right side of Eq. (4) to zero, one has

$$\cos\varphi_{m2} = -FG \pm H(G^2 + H^2 - F^2)^{1/2}/(G^2 + H^2) \tag{5}$$

where

$$\begin{aligned} F &= [(r - \cos\vartheta)/\sin\vartheta](\cos\chi \cos\epsilon - \sin\chi \sin\epsilon \cos\zeta) \\ G &= (\cos\kappa \cos\epsilon \sin\chi + \cos\kappa \sin\epsilon \cos\chi \cos\zeta - \sin\kappa \sin\epsilon \sin\zeta) \\ H &= (\sin\kappa \cos\epsilon \sin\chi + \sin\kappa \sin\epsilon \cos\chi \cos\zeta + \cos\kappa \sin\epsilon \sin\zeta) \end{aligned}$$

Since φ_{m1} is determined by the source function that is symmetric about the $\mathbf{s}$,$\mathbf{r}$ plane, the values of φ in the range $0 \leqslant \varphi \leqslant 2\pi$ which contribute to the input are $0 \leqslant \varphi \leqslant \varphi_{m1}$, $(2\pi - \varphi_{m1}) \leqslant \varphi \leqslant 2\pi$, and $\varphi_{m1} \leqslant \pi$. However, the solution of Eq. (5) for φ_{m2} yields two roots symmetric about the $\boldsymbol{\omega}$,r plane. Since the function is multivalued, the problem is to find them. For any given problem these can be determined upon careful examination of the physical picture and with the aid of the following: 1) one value φ_{m2}', say, must lie in the range $\kappa \leqslant \varphi \leqslant \pi + \kappa$, and the other, φ_{m2}'', must lie in the range $\pi + \kappa \leqslant \varphi \leqslant 2\pi + \kappa$; and 2) the values of φ which contribute to the input are $\kappa \leqslant \varphi \leqslant \varphi_{m2}'$ and $(\kappa + 2\pi - \varphi_{m2}') \leqslant \varphi \leqslant 2\pi + \kappa$. In practice, the computer program is written in such a way that it is not necessary actually to determine the limits. The only values of φ which contribute are those lying in the overlapping regions of the two ranges given by φ_{m1} and φ_{m2}. Therefore, for the φ integration, the program that has been used is written in such a way that, for each value of φ (remember that the computer calculates the function for incremental steps of φ and adds them), the computer makes a check of both $\cos\beta$ and $\cos\eta$. If *both* are $\leqslant 1$ and >0, then that value of φ contributes, and the complete computation is made and stored. If one or both is $\leqslant 0$, a zero is entered. In this manner the computer runs through the entire range of φ from 0 to 2π.

For the special case when $\chi = 0$ (i.e., the spin axis coincides with $\mathbf{r}$), the expression to be used for $\cos\eta$ is

$$\cos\eta = \frac{\sin\epsilon \sin\vartheta(\sin\zeta \sin\varphi + \cos\zeta \cos\varphi)}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}} + \frac{(r - \cos\vartheta)\cos\epsilon}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}} \tag{6}$$

Equation (6) follows from Eq. (4) by making the substitutions $\chi = 0$ and $\kappa = 0$. The latter is required because, for the case when $\chi = 0$, the angle κ has no meaning and therefore is given here the value of zero.[3] In addition, it might be well to point out here that many combinations of values of the parameters ϵ, χ, κ yield an identical physical picture as another set. Therefore, the only range of values for the parameters that need be considered is $0 \leqslant \chi \leqslant \pi$, $0 \leqslant \epsilon \leqslant \pi/2$, and $0 \leqslant \kappa \leqslant \pi$. For any value of $\epsilon > \pi/2$, the same physical picture is obtained if χ is replaced by $(\pi - \chi)$, ϵ by $(\pi - \epsilon)$, and κ by $(\pi + \kappa)$. For any value of $\kappa > \pi$, one can replace it by $(2\pi - \kappa)$, all other parameters remaining the same. The fact that a value of $\chi > \pi$ is equivalent to $(2\pi - \chi)$ is obvious.

The average over a spin period $\langle P \rangle$ defined by

$$\langle P \rangle = \frac{1}{2\pi} \int_0^{2\pi} P(\zeta) d\zeta$$

is done quite simply by the computer.

Discussion

In the foregoing, the equation for P, even though it has not been solved explicitly, is an exact expression only for the geometric aspects of the problem. The assumption that earth is a spherical, diffuse reflector is necessary if the equation is not to be much more complicated than it is. However, these assumptions would seem to be not as serious as the approximations that the reflectivity is latitude- and longitude-independent and that there is no time-dependency. This undoubtedly is not true. If the ϑ and φ dependence were known, the expression could be modified readily, resulting in essentially no additional labor for the computer integrations. The time variation is, of course, much greater, depending upon such things as cloud cover, cloud location with respect to the sun, etc. However, in assuming a certain amount of spatial uniformity (as is done in the present model), time changes can be handled because the average albedo enters the expressions only as a multiplicative constant, and the result can be changed accordingly.

References

[1] Cunningham, F. G., "Earth reflected solar radiation input to spherical satellites," ARS J. **32,** 1033–1036 (1962).

[2] Cunningham, F. G., "Power input to a small flat plate from a diffusely radiating sphere, with application to earth satellites," NASA TN D-710 (August 1961).

[3] Cunningham, F. G., "Earth reflected solar radiation incident upon an arbitrarily oriented spinning flat plate," NASA TN D-1842 (to be published). In this report, the many brief comments contained in this note [e.g., the derivation of Eqs. (4) and (6)] are covered in great detail.